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INHERITANCE IN NICOTIANA TABACUM. V. THE OCCURRENCE OF HAPLOID PLANTS IN INTERSPECIFIC PROGENIES

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When two constant species of *Nicotiana* are crossed, the hybridization, if successful, gives a uniform F_1 exhibiting unmistakable hybrid characteristics. There have, however, been sporadic reports of the production of occasional plants in interspecific hybridization in *Nicotiana* which differ from the expected hybrid type. Manifestly some of these results have been due to error, dependent particularly on the use of parent species one or both of which have not been constant; but it is difficult to explain all the reports in this way.

During the season of 1923 two plants were obtained which were outstanding exceptions to the general rule of uniformity of F_1 interspecific progenies. Both occurred in F_1 progenies of *Nicotiana Tabacum* ♀ × *N. sylvestris* ♂. As a rule this cross gives a uniform, vigorous, nearly completely sterile F_1 hybrid which is a replica *on an enlarged scale* of its particular *Tabacum* parent (cf. Goodspeed and Clausen, 1917 *a* and *b*). Two such hybrid progenies grown in 1923 gave the usual results, except that in each population one exceptional plant appeared which was a replica *on a reduced scale* of its immediate *Tabacum* parent. Both of these plants had 24 chromosomes. Since the chromosome number in *Tabacum* is $n = 24$ (White, 1913) and in *sylvestris* $n = 12$ (Goodspeed, 1923), the exceptional plants were evidently *Tabacum* haploids.

One of these exceptional plants appeared in an F_1 of *N. Tabacum* var. *purpurea* ♀ × *N. sylvestris* ♂. Among 58 plants grown, 57 were of the expected hybrid type. The exceptional plant was obviously a reduced replica of *purpurea*. It exhibited all the differential characters peculiar to *purpurea;* but the expression of the characters was somewhat exaggerated. This exaggeration of *purpurea* characters was particularly evident in the leaf base, which in *purpurea* is sharply constricted almost to the midrib.

In the haploid the constriction was more pronounced, so that the leaves had a distinct short petiole. The haploid was about three-fourths the height of *purpurea*, the leaves were smaller, the branches more slender, and the flowers distinctly smaller as is shown by the measurements recorded in the accompanying table. Since floral measurements are so remarkably constant in *Nicotiana*, this difference is of particular significance. The haploid, although somewhat less vigorous than *purpurea*, bloomed profusely, but no seeds were produced. The pollen seemed to be completely defective; the grains were exceedingly variable in size, and most of them were shrivelled and devoid of contents. Numerous attempts to secure seed by application of *purpurea* pollen failed. The capsules were retained slightly longer; but ultimately they were cast before reaching maturity. Examination of immature capsules failed to disclose the presence of developing ovules.

GARDEN NUMBERS	DESCRIPTION	NUMBER OF PLANTS MEASURED	COROLLA LENGTH	COROLLA SPREAD
23.050	*sylvestris*	50	85.3	42.5
23.014	*purpurea*	49	49.3	36.0
23.083	*purpurea* ♀ × *sylvestris* ♂	48	59.0	44.1
23.083P57	*purpurea* haploid	1	40	25
23.131	*macrophylla* derivative, normal type	23	44.8	38.0
23.132	*macrophylla* derivative × ♂ *sylvestris* ♂, normal type	27	55.1	39.9
23.132P11	*macrophylla* derivative, normal type haploid	1	39	28

The second exceptional individual was produced under somewhat less favorable conditions for demonstration. The *Tabacum* parent was a fifth generation *macrophylla* derivative obtained by back crossing an F_1 of *N. Tabacum* var. *macrophylla* ♀ × *N. sylvestris* ♂ to *macrophylla*, after which the partially fertile progeny were selfed for five consecutive generations. The derivative line was selected for an aberrant type, called "corrugated" on account of its bullated leaves, which when selfed always yields a dimorphic progeny consisting of approximately two-thirds corrugated and one-third normal plants identical in appearance and genetic behavior with *macrophylla*. From a fifth generation corrugated ♀ × *N. sylvestris* ♂ an F_1 of 50 plants was grown, 49 of which were distributed approximately equally into the two expected classes, a normal red-flowering type identical with the typical F_1 *macrophylla-sylvestris* hybrid, and a white-flowering type corresponding to the corrugated derivative. The exceptional plant was strikingly different from the plants of both of these two classes. It was a reduced replica of *macrophylla*, evidently corresponding to the normal plants of the selfed progeny of the corrugated

parent. It exhibited the same general resemblance to *macrophylla* as has been described for the *purpurea* haploid and *purpurea*. As compared with *macrophylla*, the size was somewhat reduced, the leaves were smaller and thinner, the branches more slender, and the flowers smaller as shown by the measurements in the accompanying table. The haploid was somewhat less vigorous than *macrophylla;* but it was by no means a weakling. When lifted and transferred to the greenhouse at the end of the season it continued to grow and blossom; but it set no seeds. The pollen was totally defective.

Cytological studies were made of both plants, using smears of pollen mother cells stained in aceto-carmine. Normal *Tabacum* and *sylvestris* were also examined by this method, the results corroborating those of the previous observers. Before reduction occurs the chromosomes of the haploids are the size of one member of a pair in normal *Tabacum* pollen mother cells and are split longitudinally. In fact at an early stage they greatly resemble the somatic chromosomes of the diploid, but the short rods soon form 24 compact V's which appear almost spherical, and contrast markedly with the dumb-bell shaped dyads of the diploid at the time of reduction. Reduction is a rather haphazard process, few or many of the chromosomes passing to the poles, the rest remaining at the equator of the spindle. The following distributions have been observed: $4<15>5$, $6<12>6$, $7<12>5$, $9<6>9$, $8<11>5$, $7<10>7$, $8<10>6$, $8<1>15$. In one case a single giant spindle contained all 24 chromosomes. The chromosomes which remain at the equator often divide. The second division is somewhat more regular than the first, but lagging chromosomes frequently fail to reach either daughter nucleus, forming extra small nuclei which later form microcytes. The pollen mother cells divide to form 2–5 cells which differ greatly in size, except when two cells result. The latter may be the result of non-reduction; but if this is the case, a certain amount of normal *Tabacum* pollen should be present.

The haploid tobaccos resemble the haploid Daturas (Blakeslee, et al, 1922), in the random assortment of whole chromosomes at the first division, the formation of two pollen cells as a result of the division of certain pollen mother cells, and in the high percentage of bad pollen which they produce. The appearance of an occasional haploid plant may account for the production in some cases of sterile, maternal plants in addition to true hybrids in interspecific F_1 progenies.

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ELECTRODYNAMICS IN THE GENERAL RELATIVITY THEORY

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Several attempts have been made to include the electrical phenomena in the general relativity theory by substituting a more general space for the Riemann space (Weyl, Eddington; the expression for the Riemann tensor as dependent on the electromagnetic tensor suggested in a previous paper of the author[1] also requires a generalization of geometry). But perhaps this generalization is after all not necessary and it may be possible to include electromagnetism in the general relativity theory without breaking the frame of the Riemann geometry. This note deals with an attempt to work out some consequences of what may be considered as pretty well established relations.

1. We hold fast to the idea of the previous note that the curvature tensor must be determined by the electromagnetic tensor but instead of putting down an *a priori* expression we will start with the known energy relation

$$F_{ij} = \tfrac{1}{2}\{f_i^k f_{kj} - d_i^k d_{kj}\} \qquad (1)$$

where F_{ij} is the contracted curvature tensor, f_{ij} the electromagnetic tensor and d_{ij} its dual; this relation does not give the explicit expression for the curvature, it is true, but we will try to obtain from it as much information as possible. Together with equation (1) we have to consider the Maxwell equations

$$f_{i,j}^j = 0, \qquad d_{i,j}^j = 0. \qquad (2)$$

2. There are two points of view from which the system (1)–(2) can be considered. If we concentrate our attention mainly on the Maxwell equations (2) we may remark that the existence of the relation (1) deprives them of their linearity: in fact, if we consider the f_{ij} as the unknowns and try to introduce them in the second set of the equations (2) we notice that the d's can be expressed through the f's only with the aid of the components of the fundamental tensor g_{ij} and since the g's enter in the left part of (1) they depend on the f's so that the second set of (2) is not linear

in the f's, or *the whole system* (1)–(2) *is not linear with respect to the electromagnetic tensor*. This remark shows that the so-called classical electrodynamics cannot be transferred without alterations into the general relativity theory and that it is only an approximation which is, in some cases, admissible at considerable distances from the regions of intense electromagnetic field where we can neglect the right part of (1) and, therefore, treat (2) independently of (1); the application of "classical electrodynamics" in atomic regions contradicts thus not only experience but also the general relativity theory and it is, therefore, to be hoped that the rigorous solution of the whole system (1)–(2) will lead us toward a theory of atomic processes on the basis of the general relativity theory. From this point of view the elementary singularities of fields satisfying (1)–(2) seem to be of the greatest interest but the treatment of this system seems to be very difficult just on account of its non-linearity. That is why the second point of view introducing geometrical considerations may be of some use.

3. If we focus our attention on equation (1) and consider the (2) as supplementary conditions we may present our problem in the following form: find a space with a curvature tensor $F_{ij,kl}$ and in this space an antisymmetrical tensor f_{ij} which satisfies the relation (1) and also the Maxwell equations. We will ask ourselves what conditions are imposed on the space by the existence of such a tensor.

We mentioned in the previous paper[1] that an antisymmetrical tensor or linear vector function $f(x)$ transforms vectors of each of two absolutely perpendicular planes into vectors belonging to the same planes, respectively. It will be necessary now to write down the explicit expression of f; if i, j and k, l denote two couples of perpendicular unit vectors which belong, respectively, to these two planes the expressions of $f(x)$ and its dual are

$$\begin{aligned} f(x) &= \lambda\{i(jx) - j(ix)\} + \mu\{k(lx) - l(kx)\}, \\ d(x) &= \mu\{i(jx) - j(ix)\} + \lambda\{k(lx) - l(kx)\}. \end{aligned} \tag{3}$$

We will discriminate between the *skeleton* of the antisymmetrical function, which is given by the two planes, and the function itself, which is given if we give in addition to the skeleton the two numbers λ and μ. We consider now a region in each point of which an antisymmetrical function is given; we will say in this case that we have a field and we will call the totality of the skeletons of the functions which constitute the field the *skeleton of the field*. We can choose the couples i, j and k, l in their respective planes in such a way as to make $i.j'(h) = 0$, $j.i'(h) = 0$, $k.l'(h) = 0$, $l.k'(h) = 0$ (where $i'(h)$ denotes the differential of i, etc.) and we will suppose that this choice is made. In connection with the skeleton there can be considered in each point two vectors

$$p = i.divi + j.divj + k.divk + l.divl \tag{4}$$

and

$$q = i.\{k'(l).j - l'(k).j\} + j.\{l'(k).i - k'(l).i\} + k\{i'(j).l - j'(i).l\} + l\{j'(i).k - i'(j).k\}. \quad (5)$$

It can be proved that the necessary and sufficient condition for the possibility of constructing over a skeleton a field which satisfies the Maxwell equations is that the vectors p and q *be gradients of scalar functions;* and it may be of interest to note that, for a Maxwellian field, the *vanishing* of the vector q is the necessary and sufficient condition for the possibility of considering the planes of the skeleton as tangent planes of two families of surfaces.[2]

4. We return now to the equation (1) which we write in vector form

$$F(x) = \tfrac{1}{2}\{f^2(x) - d^2(x)\}; \quad (6)$$

substituting the expressions (3) we find

$$F(x) = \frac{\mu^2 - \lambda^2}{2}\{i(ix) + j(jx) - k(kx) - l(lx)\}. \quad (7)$$

This shows that $F(x)$, i.e., the contracted curvature tensor, is a linear vector function *of a special kind;* geometrically we can say that it has not only four mutually perpendicular invariable directions (as every symmetrical linear vector function) but two (absolutely perpendicular) planes each direction of which is invariable; algebraically we can say that the secular equation formed with the coefficients F_{ij} has two positive and two negative roots all of the same absolute value. These are the conditions imposed on the space *by the antisymmetricity* of the electromagnetic tensor. We see that if these conditions are fulfiled the space itself determines a skeleton and we can consider the two vectors p and q which correspond to this skeleton; the fact that the electromagnetic tensor *satisfies the Maxwell equations* (2) requires now that these vectors be gradients. It can be shown that p is always a gradient; that is a consequence of the Hilbert equations $\partial(F_i^j - \frac{1}{2}\delta_i^j F)/\partial x^j$ which are an immediate consequence of the Bianchi equations (3) of the paper[1] and, therefore, are satisfied in every Riemann space. As for the condition that q be a gradient it seems, in the contrary, not to be a consequence of the general properties of curved space and to impose, therefore, an additional condition on the space. If this condition is fulfiled an antisymmetrical tensor satisfying the equations (1) and (2) can always be found.

5. In looking for spaces where q is a gradient it is natural, following the idea of Kasner,[3] to start with the consideration of a four-dimensional space which can be imbedded in a five-dimensional flat space. This case can be treated in much the same manner as the case of a two-dimensional surface in a three-dimensional space;[4] the curvature conditions are controlled by a linear vector function $s(x)$ which satisfies the Codazzi equation

$$s'(x,h) = s'(h,x), \quad (8)$$

the Riemann tensor has the same expression in s as in the simpler case (comp. form. (11) of the paper cited under[4]); it is easy to see that in the case we are interested in $s(x)$, like $F(x)$, has two pairs of equal roots of the secular equation and the same planes of invariant directions as $F(x)$, and the Codazzi equation has for its consequence the *vanishing of both vectors p and q;* since the vanishing is a particular case of being a gradient, a Maxwellian field can be extended over the skeleton but it is a very particular case of the general Maxwellian field since both invariants of the electromagnetic tensor are constant throughout the field.

The problem to find all the spaces of the type considered in this section is analogous to the problem to find all the Weingarten surfaces of a given type; using the representation with the aid of the generalized stereographical projection[5] it can be reduced to the finding of one scalar function of four variables which satisfies three partial differential equations of the second order (not linear).

It seems to be natural to look for further solutions in the form of four-dimensional space imbedded in a six-dimensional flat space.[6]

[1] These PROCEEDINGS, **9**, No. 12, 1923, p. 405.

[2] Comp. Whittaker, E. T., *Edinburgh, Proc. R. Soc.*, **42** (1–23).

[3] Baltimore, *Amer. J. Math.*, **43**, 1921 (126–129).

[4] These PROCEEDINGS, **9**, No. 6, 1923 (179–183); a more detailed presentation to appear in the *Amer. J. Math.*, **46**.

[5] These PROCEEDINGS, **9**, No. 12, 1923 (401–403).

[6] See papers by Kasner, *Amer. J. Math.*, **43**, 1921 (130–133), and (217–221); *Science*, **54**, 1921 (304); *Math. Ann. Leipzig*, **85**, 1922 (227–236).

PRINCIPAL DIRECTIONS IN AN AFFINE-CONNECTED MANIFOLD OF TWO DIMENSIONS

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1. None of the principal directions defined in a previous paper (these PROCEEDINGS, **8**, No. 7, p. 198) exist in a manifold possessing components of affine connection (Γ^i_{jk}) without a fundamental tensor (g_{ij}). In the present paper two types of principal directions are defined in an affine-connected two-space; in the case where the space is Riemannian, by which we understand the existence of a fundamental tensor g_{ij} such that

$$\left\{ {j\ k \atop i} \right\} = \Gamma^i_{jk}, \tag{1.1}$$

it will be shown that these principal directions reduce to the null-directions.

By "principal directions" are understood directions defined by invariant relations.

2. In an affine-connected two-space the equations

$$F^i_{jkl} = \frac{\partial \Gamma^i_{jl}}{\partial x^k} - \frac{\partial \Gamma^i_{jk}}{\partial x^l} + \Gamma^i_{rk}\,\Gamma^r_{jl} - \Gamma^i_{rl}\,\Gamma^r_{jk}, \tag{2.1}$$

$$F_{jl} = F^i_{jil}, \tag{2.2}$$

define the curvature tensor and the contracted curvature tensor, respectively. The equation

$$F_{ij}\xi^i\xi^j = 0 \tag{2.3}$$

is invariant: we shall call the directions ξ^i so defined the *principal directions of Type I*.

If a vector ξ^i be propagated parallelly around an infinitesimal circuit of parametric lines of a coördinate system (u, v), and the limit of the increments in its components, divided, by the product of the increments in u and v which define the circuit, be denoted by η^i, then

$$\eta^i = -F^i_{jkl}\xi^j \frac{\partial x^k}{\partial u}\frac{\partial x^l}{\partial v}, \tag{2.4}$$

and the direction defined by η^i is independent of the coördinate system (u, v) (*Annals of Mathematics*, Dec., 1923). We may, therefore define as *principal directions of Type II* the directions of those vectors which regain their original directions after propagation around an infinitesimal circuit. The equations for such directions are

$$\eta^i = \theta\xi^i, \tag{2.5}$$

where θ is indeterminate. Choosing $u = x^1$, $v = x^2$, we obtain

$$\theta\xi^i + F^i_{j12}\xi^j = 0, \tag{2.6}$$

from which the equation for θ is

$$\begin{vmatrix} \theta + F^1_{112} & F^1_{212} \\ F^2_{112} & \theta + F^2_{212} \end{vmatrix} = 0 \tag{2.7}$$

3. If the two-space is Riemannian, we have

$$F^i_{jkl} = g^{ri}\,F_{rjkl}, \tag{3.1}$$

where

$$F_{ijkl} = -F_{jikl} = F_{klij}. \tag{3.2}$$

Thus (cf. Laue: *Relativitätstheorie* Bd., 2, p. 98)

$$F_{jl} = g^{ri}\,F_{rjil} = \tfrac{1}{2}g_{jl}\,F, \tag{3.3}$$

where

$$F = g^{ij}\,F_{ij}. \tag{3.4}$$

Thus the directions of Type I are given by

$$F.g_{ij}\xi^i\xi^j = 0, \tag{3.5}$$

or, if F is not zero, by the null-directions.

If equations (2.6) are multiplied by $g_{ik}\xi^k$ and summed, the directions of Type II are given by

$$\theta g_{ik}\xi^i\xi^k + F_{kj12}\,\xi^j\xi^k = 0. \tag{3.6}$$

But from (3.2),

$$F_{kj12}\,\xi^j\xi^k = 0, \tag{3.7}$$

and the directions are given by

$$\theta g_{ik}\xi^i\xi^k = 0. \tag{3.8}$$

We find

$$\begin{cases} F^1_{112} = \frac{1}{2}g_{12}\,F, & F^1_{212} = \frac{1}{2}g_{22}\,F, \\ F^2_{112} = -\frac{1}{2}g_{11}\,F, & F^2_{212} = -\frac{1}{2}g_{12}\,F, \end{cases} \tag{3.9}$$

and the equation (2.7) for θ becomes

$$\theta^2 + \tfrac{1}{4}gF^2 = 0. \tag{3.91}$$

Thus, if neither F nor g is zero (3.8) gives the null-directions.

A STATISTICAL DISCUSSION OF SETS OF PRECISE ASTRONOMICAL MEASUREMENTS: PARALLAXES

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In discussing the precision of the determination of the velocity of light Simon Newcomb remarked,[1] "So far as could be determined from the discordance of the separate measures the mean errors of Newcomb's result would be less than 10 km., but making allowance for the various sources of systematic error the actual probable error was estimated at ±30 km." Thus did Newcomb give expression to a belief that the probable error as calculated from the data was likely to represent only a fraction of the real probable error connected with the determination of a mean, and although he would have liked to claim the maximum precision for his value of the velocity of light, and probably did claim all he thought safe, his judgment led him to give as the actual probable error the triple of that calculated by his formula. If Newcomb had had faith in this value ±10 km. he would have asserted that the chance of the departure of the velocity of light from his mean by so much as 30 km. was less than one in twenty-two; instead, he said not more than one in two.

In view of the ever widening application of statistical methods to fields of investigation less precise, less controlled, and less controllable than the

best physical experimentation, and in view of the tendency manifest in some quarters to place a great deal of confidence in the probable errors for mean values, derived by the application of mathematical formulas, there may be some value in discussing the question of how far an actual probable error may exceed the value calculated from the discordance of the separate measures. The discussion may be carried out theoretically, i.e., mathematically, by an analysis of the derivation of the formula for the probable error, or experimentally by the examination of the actual determined values of physical measurements and their probable errors. The latter method is the one selected here, and in applying it we naturally turn first to the sort of measurements in which precision is most important, and has been striven for by every known means: determination of parallax.

The modern observations of trigonometric parallaxes may undoubtedly be classed among the most precise in astronomy. Parallaxes have been

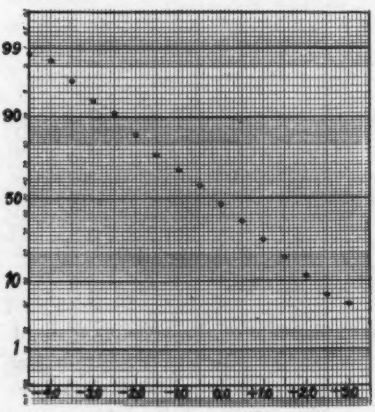

FIGURE 1

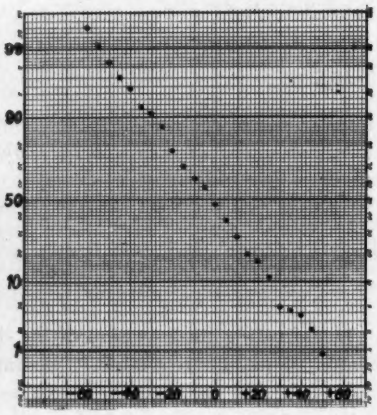

FIGURE 2

Fig. 1. Plot on arithmetic-probability paper. Ordinates represent percentages of stars for which the quantity α explained in the text exceeds the value given by the abscissae.

Fig. 2. Plot on arithmetic-probability paper. Ordinates represent percentages of stars for which the difference in parallax. (Allegheny relative + 0".001 − McCormick relative) exceed values given by abscissae. (Unit 0".001.)

determined by photographic methods, employing refractors or reflectors of long focal length, at the Allegheny, McCormick, Greenwich, Mt. Wilson, Yerkes, Sproul and Dearborn observatories. Of all the published results, only those of Allegheny and McCormick have a sufficient number of stars in common to allow a really satisfactory statistical discussion.

Let us first consider what results we may expect. Both observatories (Allegheny and McCormick) determine their parallaxes by measuring,

at different epochs, the position of the parallax star relative to three or four comparison stars. The basis on which these stars are selected is as follows: 1° they should be reasonably close, and evenly distributed around the central parallax star, and 2° they should be of approximately the same photographic brightness as the reduced image of the comparison star. It therefore appears not unlikely that there may have been considerable overlapping in the stars used by both observatories (Allegheny and McCormick), and that a part of the systematic error arising from the comparison stars will be the same for both. There seems to be no *a priori* reason for assuming that any of the experimental technique would introduce a negative correlation between the results, and accordingly we may expect a positive, though probably small, correlation between the two series, which will result in the probable error of the difference between the values of the individual parallaxes being less than the root of the sum-square probable error of the two:

$$r_{A-M} = \sqrt{r_A^2 + r_M^2 - 2\rho r_A r_M}, \quad \rho = \text{coefficient of correlation.} \quad (1)$$

If we take for each star the difference $d\pi$ = parallax Allegheny − parallax McCormick, and divide by the quantity $r = \sqrt{(r_A^2 + r_M^2)}$ calculated from the probable errors reported, we should expect that the resulting series of values $d\pi/r = \alpha$ should be distributed in an error curve, and that the parameter of this curve should be such as to make a trifle more than one-half of the values of α fall between $\alpha = \pm 1$. Or rather, this is what we must expect if we believe that the actual probable errors are those deduced from the discordances of the observations in the individual parallax determinations.

We have 273 values of α at our disposal.[2] As it appears from other investigations that the reduction from relative to systematically correct absolute parallax is $+0''.004$ for Allegheny and $+0''.003$ for McCormick, $d\pi$ was then taken to be 0.001 larger than the difference between the relative Allegheny and McCormick values.

The distribution curve for the quotient α, plotted on arithmetic-probability paper is shown in figure 1. The accordance with the normal error curve is perfect. The probable error of the present curve (the probable error is 0.6745 σ if σ is the standard deviation of the curve $e^{-\alpha^2/2\sigma^2}$: $\sqrt{2\pi}\,\sigma$) is, however, decidedly larger than unity, viz: 1.24 ± 0.06. The frequency of the difference $d\pi$ itself also follows a normal error curve, as is shown in figure 2, with $-0''.0023$ as mean value, and $\pm 0''.015$ as its probable error.

We conclude that, on the whole, the McCormick parallaxes are $0''.0033 \pm 0''.0010$ larger than the Allegheny values, and that so far as may be judged by this comparison of the two sets of values, the actual probable errors of the individual determinations are on the average about 25% greater than those calculated from the observations.

If we now return to the general formula (1) and ask what coefficient of correlation ρ would explain the spread of the values of α as found, we have:

$$r_{A-M} = \sqrt{(r_A^2 + r_M^2 - 2\rho r_A r_M)} = 1.24\sqrt{(r_A^2 + r_M^2)}$$

Hence

$$\rho = .27\,(\gamma_A/\gamma_M + \gamma_M/\gamma_A) = -.54\ (\pm.15)$$

It will probably not seem so reasonable to attribute the extra dispersion to such a high negative correlation between the observations as to a relatively small systematic underestimation of the probable errors when calculated by the rule.

It is a testimonial to the judgment with which parallaxes determinations are reduced that the factor of amplification in passing from the calculated to the actual probable errors in this series of 237 Allegheny-McCormick values should be as low as 1.24. There are possibilities of correlation between these series that may have reduced this value; there is the probability that if the same series of stars were observed by a large number of observatories using a variety of methods we should have evidence of a somewhat larger factor, say 1.5, but the evidence is reasonably good that we should not have to use a figure anywhere nearly as high as the 3 suggested by Newcomb for his determination of the velocity of light.

Of the other parallax series only Yerkes (Ye) has a sufficiently large number of stars in common with Allegheny (A) and McCormick (M) to be treated with success. The differences A−Ye and M−Ye were first obtained, and it was found that their arithmetical mean values were: A − Ye = − 0".0017 ±0".0022 (82 stars); M−Ye = − 0.0042 ±0.0020 (87 stars). After correcting the $d\pi$ for these systematic differences the respective α's were formed. In both cases the distribution curve of α was as nearly normal as could be expected from such scanty material. It was found that the probable error of an individual difference A−Ye was 1.51 ±0.12 larger, and that of M−Ye was 1.32 ±0.10 larger than the resultant of the published probable errors.[3]

[1] Encyclopedia Britannica, **16,** p. 625 (11th edition).

[2] Based on all published parallaxes and some unpublished values kindly put at our disposal by Dr. Curtis and Dr. Mitchel. For results based on 56 stars see van Rhijn, *Publ. Astron. Lab. Groningen,* 1923, No. 34, p. 74.

[3] The three differences *A-M, M-Ye,* and *Ye-A* do not add up to zero, in spite of the fact that some stars are common to all three series. They must, therefore, be considered to have significance for our work only insofar as their application reduces the values of the ratios between the presumptive and the published probable errors to a minimum.

THE QUANTUM THEORY OF THE FRAUNHOFER DIFFRACTION

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1. *Introductory.*—In an important paper published on the pages of these PROCEEDINGS, W. Duane[1] makes a successful "attempt to formulate a theory of the reflection of X-rays by crystals, based on quantum ideas without reference to interference laws." A. H. Compton,[2] enlarging upon a hint contained in Duane's paper, has recently pointed out that the latter's hypothesis can be justified by the application of the general rules of the theory of quanta to the translatory motions of a crystal lattice.

Both these authors are dealing with the case of parallel beams of incident and reflected light (Fraunhofer diffraction) and of infinite lattices giving absolutely sharp pencils of reflected rays of different order. The purpose of the following lines is to study the problems with respect to finite gratings and other diffracting systems, again with restriction to the case of the Fraunhofer reflection.[3]

A finite grating, as the most extreme case of which we can regard the totality of only two reflecting points, can be regarded as a superposition of infinite gratings according to Fourier's theorem. Mathematically, therefore, our problem is reduced to a Fourier analysis. On the other hand, a finite grating produces a more or less continuous spectrum, and in the quantum theory the intensity corresponding to any given angle of deflection of the rays by the grating must be expressed in terms of the probability of the light quanta undergoing such a deflection. These circumstances suggest *the principle of correspondence* as a suitable instrument for carrying through our investigation.

2. *The Duane-Compton Rules of Quantization.*—We state here the Duane-Compton rules in a generalized form bringing out the invariancy of the result for any choice of axes. Let us consider a three dimensional infinite triclinic lattice with the spacings a_1, a_2, a_3 in the respective directions of its chief axes. The contention of Compton is that in a collision with a light quantum such a lattice can only pick up a linear momentum the orthogonal projections of which p_1, p_2, p_3 on the directions q_1, q_2, q_3 of the chief axes satisfy the fundamental conditions of the quantum theory

$$\int p_1 dq_1 = n_1 h, \qquad \int p_2 dq_2 = n_2 h, \qquad \int p_3 dq_3 = n_3 h \tag{1}$$

n_1, n_2, n_3 are three integral numbers and h denotes Planck's constant of action. The periodicity of the lattice is given by its spacings a_1, a_2, a_3 so that the first integral is to be extended from q_1 to $q_1 + a_1$, and the others correspondingly. We obtain, therefore,

$$p_1 = hn_1/a_1, \quad p_2 = hn_2/a_2, \quad p_3 = hn_3/a_3. \tag{2}$$

On the other hand the momentum of a light quantum of the frequency ν is given by $h\nu/c = h/\lambda$, where c is the velocity of light and λ the wave-length in vacuo corresponding to the frequency ν. If the direction of propagation of the light includes with its main axes angles the cosines of which are, before its collision with the lattice, α_0, β_0, γ_0, and after its reflection α, β, γ, each light quantum loses in the collision a momentum the projections of which on our main axes are $h(\alpha - \alpha_0)/\lambda$, $h(\beta - \beta_0)/\lambda$, $h(\gamma - \gamma_0)/\lambda$.

The principle of conservation of momentum requires, therefore, the relations

$$\alpha - \alpha_0 = \lambda n_1/a_1, \quad \beta - \beta_0 = \lambda n_2/a_2, \quad \gamma - \gamma_0 = \lambda n_3/a_3, \tag{3}$$

which are identical with those derived by von Laue from the theory of interference.

However, the choice of the main axes is arbitrary and we can remove this arbitrariness only by proving that if the lattice acquires in a collision a momentum in the direction of any straight line connecting two of its points (following Bragg we shall call such a direction a "crystal avenue"), this momentum must satisfy a condition analogous to (2)

$$p.a = nh \tag{4}$$

where a is the smallest distance between two points in that line. Referred to the main axes, a will be the d agonal of a parallelepipedon with the edges m_1a_1, m_2a_2, m_3a_3, if m_1, m_2, m_3 are three integers prime to each other. It is easy to see that in terms of these edges

$$a = m_1a_1 \cos(a_1a) + m_2a_2 \cos(a_2a) + m_3a_3 \cos(a_3a)$$

Applying conditions (2) we get

$$p \cos(a_1a) = \lambda n_1/a_1, \quad p \cos(a_2a) = \lambda n_2/a_2, \quad p \cos(a_3a) = \lambda n_3/a_3.$$

Multiplying the three equations by m_1a_1, m_2a_2, m_3a_3, respectively, and adding them we receive

$$pa = (m_1n_1 + m_2n_2 + m_3n_3)\lambda$$

As m_1, m_2, m_3 are prime to each other, the parenthesis, according to a well known theorem of theory of numbers, by a suitable choice of positive or negative integral values for n_1, n_2, n_3, can be made equal to any integer n.

We see, therefore, that relation (4) is contained in our conditions (2) or, in other words, that the latter conditions may be imposed on the orthogonal projections of the momentum on any three crystal avenues.

3. *Infinite Linear Grating.*—Let us begin our considerations with the single case of a one dimensional grating the elements of which are arranged in a straight line. In this line (the distance in which from a fixed point we denote by x) the material points with which the light quanta may collide are distributed with a certain density ρ. We shall call ρ for short

the "*electronic density*," and the notion of a grating involves that this electronic density is a periodic function of x with a period a called "the spacing of the grating."

If the grating is moving at a constant velocity in the direction x relatively to a resting point in this line, the density in that point will change as a periodic function of the time and return to its original value after the grating has moved through the distance a or a multiple of it. That is the reason why Compton regards the spacing a as the region over which the quantum integral $\int pdq = nh$ must be extended (cf. section 2), giving the relation

$$p = hn/a \tag{4}$$

If the distribution of electronic density was a sinusoidal one represented by the formula

$$\rho = A \sin (2\pi x/a + \delta) \tag{5}$$

the change of density in a fixed point, due to the motion of the grating, would be a simple harmonic oscillation. By means of the Fourier theorem any distribution of electronic density can be built up of sinusoidal terms, in other words, any grating, infinite or finite, can be represented as a superposition of infinite sinusoidal gratings of the type (5). This case deserves, therefore, a particularly close study. The principle of correspondence tells[4] us that to every harmonic term in the expression of ρ there corresponds a quantum change of motion accompanied by a change of momentum p given by our equation (4) if we substitute in it for n/a the coefficient of $2\pi x$ in the argument of the sine.

If, therefore, the Duane-Compton relation[4] tells us that a grating can only pick up momentum in multiples of the quantity h/a, the principle of correspondence permits us to go farther and to say that a *sinusoidal grating of the constitution* (5) *will experience only changes of momentum in amounts* $\pm h/a$ *and not in multiples of it.*[5]

The general expression for ρ in an infinite grating is

$$\rho = \sum_0^\infty n \; A_n \sin(2\pi nx/a + \delta) \tag{6}$$

To a term of this series with the coefficient n/a of the argument $2\pi x$ there corresponds a change of momentum given by the same value of the coefficient of h in equation (4). In a lattice of such a constitution momentum can be, therefore, picked up in a large variety of ways. Moreover, the principle of correspondence gives us additional information with respect to the relative frequency of the different possible changes of momentum: the probability of the grating's picking up the momentum nh/a is proportional to the square of the coefficient of the corresponding term of our series, that is to A_n^2.

As stated in section 2 the change of momentum experienced by a grating in its collision with a light quantum determines the direction of emergence of the light quantum according to the equation

$$\alpha - \alpha_0 = \lambda n/a. \tag{7}$$

The above statement on the probabilities of momenta means, therefore, that the intensity of the spectrum of the n^{th} order will be proportional to A_n^2.

In order to show that these conditions are in agreement with the interference theory of gratings we have only to prove that a sinusoidal grating produces the same effect from the point of view of the latter theory, that is that a distribution of electronic density represented by the formula

$$\rho = A_m \sin 2\pi m x/a \tag{8}$$

will give two absolutely sharp reflected beams at angles following from (7) by putting $n = \pm m$ with intensities proportional to A_m^2. The proof is easily given: An element dx of the grating gives a contribution to the amplitude of light, emitted in a direction α, which is proportional to the modulus of the expression

$$\rho e^{i\frac{2\pi}{\lambda}x(\alpha-\alpha_0)}dx \tag{9}$$

The total amplitude in that direction is, therefore

$$S = C.A_m \int \sin 2\pi m \frac{x}{a}.e^{i\frac{2\pi}{\lambda}x(\alpha-\alpha_0)} \tag{10}$$

We shall evaluate this expression first for a finite grating taking as limits of integration $\pm Na$ and then go over to $N = \infty$.

$$S = \frac{iCA_m}{2\pi}\left\{\frac{\sin\left(\frac{m}{a}-\frac{\alpha-\alpha_0}{\lambda}\right)Na}{\frac{m}{a}-\frac{\alpha-\alpha_0}{\lambda}} - \frac{\sin\left(\frac{m}{a}+\frac{\alpha-\alpha_0}{\lambda}\right)Na}{\frac{m}{a}+\frac{\alpha-\alpha_0}{\lambda}}\right\}.$$

We see that the amplitude is proportional to A_m and that it has two maxima in the two directions $\alpha - \alpha_0 = \pm m\lambda/a$. Moreover, the maximum amplitude is proportional to N, and when N becomes infinite the intensity in the maximum completely dominates so that the whole energy is thrown into the directions of the maxima and the latter become absolutely sharp. There is, therefore, a complete identity in the Fraunhofer diffraction produced by an infinite sinusoidal grating from the point of view of the classical theory and from the point of view of the theory of light quanta sketched above. As any linear diffracting system can be built up by infinite sinusoidal gratings this identity will hold for the totality of all phenomena of Fraunhofer reflection. *The considerations of the first half of this section contain, therefore, the complete translation of the theory of Fraunhofer diffrac-*

tion into the language of the quantum theory. The following section contains only the application of these principles to one or two special cases.

4. *The linear Point Lattice.*—From the mathematical point of view the sinusoidal grating treated in the preceding section is the simplest. However, such a grating cannot be realized physically, because in some points its density becomes negative producing a reflection connected with a change of phase by half a period. The grating that is considered as the fundamental one in most text books is the *point grating:* the graphical representation of the electronic density ρ of such a grating as a function of x being a succession of equidistant peaks, very narrow compared with the spacing a.

Infinite Point Lattice.—Analytically we can express the distribution in this case by a Fourier series. Computed in the well known way, the coefficients of all the terms of lower order turn out to be the same. If C is the height of a peak and c its breadth we get for ρ the series (6) with $A_0 = Cc/a$, $A_n = 2Cc/a$. *This means that the spectra of different order produced by such a grating are all of the same intensity,* as the intensity of the n^{th} order is proportional to A_n^2. It need not disturb us that the above expressions for A_n do no longer hold for large numbers n, because the corresponding terms have no physical significance: the corresponding change of momentum, though theoretically possible, will never take place because the impinging light quantum does not possess enough momentum to realize it.

Finite Point Lattice.—In this case we have to use the Fourier integral instead of the Fourier series. If we choose as origin ($x = 0$) the centre of the grating, the expression for the electronic density will be

$$\rho(x) = \int_0^\infty A(\omega) \cos 2\pi\omega x d\omega \qquad (11)$$

$$A(\omega) = \frac{2}{\pi}\int_{-\infty}^{+\infty} \rho(\beta) \cos 2\pi\omega\beta \, d\beta \qquad (12)$$

This grating appears, therefore, as a superposition of an infinite number of sinusoidal gratings with the respective spacings $a' = 1/\omega$. According to section 3 such a grating will produce a reflected ray in a direction given by the relation

$$\alpha - \alpha_0 = \lambda/a' = \omega\lambda, \qquad (13)$$

while the relative intensity of this ray is given by the square of $A(\omega)$. This intensity is easily computed from (12): as $\rho(\beta)$ is different from zero only in the positions of the peaks $\beta = ma$, we get

$$A(\omega) = \frac{2}{\pi}\frac{Cc}{a}\sum_{-U}^{+U} \cos 2\pi m\omega a = \frac{2}{\pi}\frac{Cc}{a}\frac{\sin N\pi a\omega.}{\sin \pi a\omega}$$

if we denote by $N = 2U + 1$ the total number of peaks in our grating. To express the intensity produced by the grating as a function of the direction we have only to substitute into the square of A the expression of ω from equation (13)

$$A^2 = 4\frac{C^2c^2}{\pi^2a^2}\frac{\sin^2 N\pi a(\alpha-\alpha_0)/\lambda}{\sin^2 \pi a(\alpha-\alpha_0)/\lambda}, \tag{14}$$

in complete agreement with the classical interference formula.

In the special case, when $N = 2$, we obtain from (14) the well known distribution due to the interference of two dipoles

$$A^2 = 16\cos^2 \pi a(\alpha-\alpha)_0/\lambda. \tag{15}$$

5. *The Space Lattice.*—The generalization for the three-dimensional case does not involve any new ideas. If we denote the three crystal avenues of a triclinic lattice chosen as main axes by x_1, x_2, x_3, the distribution of electronic density in any lattice of this type and, in fact, in any other system can be built up of terms of the type

$$\rho_{m_1m_2m_3} = A_{m_1m_2m_3}\sin\left(2\pi m_1\frac{x_1}{a_1}+\delta_{m_1}\right)\sin\left(2\pi m_2\frac{x_2}{a_2}+\delta_{m_2}\right)$$
$$\cdot\sin\left(2\pi m_3\frac{x_3}{a_3}+\delta_{m_3}\right) \tag{16}$$

therefore it is sufficient to discuss the distribution of density given by this equation.

Applying the principle of correspondence in the same way as in the case of the linear grating, we conclude that such a lattice can pick up only a momentum the orthogonal projections of which on the directions x_1, x_2, x_3 are given by equations (2) with $n_1 = \pm m_1$, $n_2 = \pm m_2$, $n_3 = \pm m_3$. According to the analysis of such a motion in section 2 this means that our lattice can only acquire momentum directed in one of the four crystal avenues with the oblique components $\pm m_1a_1$, $\pm m_2a_2$, $\pm m_3a_3$ and in each of these directions with only one definite velocity (both ways, positive and negative).

The direction of the light quantum after the collision, is given by equations (3). As α, β, γ are not independent, we see that a collision can only occur if λ satisfies the von Laue-Bragg condition with absolute sharpness. In this case the lattice will give us with equal probability eight different directions of the reflected rays.

In order to prove the complete equivalence of the quantum theory with the classical treatment for three dimensional distributions, we have only to show that the interference theory leads to the same results in the case of distribution (16). In the classical theory the mean amplitude in a direction α, β, γ is proportional to the modulus of the expression

$$\iiint \rho e^{i\frac{2\pi}{\lambda}[x_1(\alpha-\alpha_0)+x_2(\beta-\beta_0)+x_3(\gamma-\gamma_0)]}dx_1dx_2dx_3. \qquad (17)$$

Introducing for ρ the distribution (16) we see that expression (17) is the product of three factors of the type given by formula (10) and discussed in section 3. It follows from that discussion that the whole reflected energy will be thrown into the same eight directions which were found on the basis of the quantum theory. This proves in a general way the complete identity of results in both treatments for any possible system, and it is not necessary to enter into special examples.

6. *Conclusion.*—The above considerations are restricted to the case of the Fraunhofer diffraction and neglect the small change of the wave-length due to the Compton effect. Moreover they are dealing only with the linear momentum without reference to the possible changes of angular momentum and other quantic conditions of the system. The last restriction seems natural as only the linear momentum has a direct connection with the direction of motion of the light quantum which is the only important element of our discussion. On the contrary the restriction to Fraunhofer phenomena does not appear to be a necessary one and we hope to extend our theory to more general cases.

The situation in optics appears, therefore, to be thus:

1. The photoelectric phenomenon and the Compton effect can be explained only by the action of light quanta.

2. The phenomena of Fraunhofer diffraction can be treated as well on the basis of the wave theory of light as by a combination of the concept of light quanta with Bohr's principle of correspondence.

3. The phenomena of coherence resist all attempts of the quantum theory.

However, it must be remembered that Bohr's principle of correspondence contains the essential features of the wave theory in a form suitable for the quantum theory. Our treatment, therefore, means rather a readjustment than a complete abandoning of the wave theory.

[1] W. Duane. *Proc. Nat. Acad. Sci., Washington*, 9, p. 159 (1923).

[2] A. N. Compton. *Ibid.*, 9, p. 359 (1923).

[3] Moreover we neglect the slight changes of the wave-length due to the Compton effect.

[4] Strictly speaking the principle of correspondence must be applied to the totality of the grating and the incident light wave, because without an exciting wave the uniform motion of a grating does not produce any radiation.

[5] We have to include the sign minus because (5) can be written also $\rho = -A \sin(-2\pi x/a - \delta)$.

EFFECTIVE RADII OF GAS MOLECULES

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When electrons pass through a gas and produce ions, the number of ions produced is directly proportional to the gas pressure, to the number of electrons passing through the gas and to the distance which the electrons travel through the gas. It is also found that with all these factors constant, the number of ions produced decreases with increasing velocity of electrons. If the effective molecular radius, is defined as the radius of a sphere about the molecular center, within which (on the average) an electron must pass to ionize the gas molecule, this decrease in ionization may be interpreted as a decrease of the effective molecular radius.

The change in ionization as a function of the velocity of the ionizing electrons is the subject of the present investigation. The apparatus used was very similar to that described by Barber[1] and McAllister,[2] and used by them to study secondary electron characteristics. The apparatus consisted essentially of a central tungsten filament surrounded by three concentric cylindrical electrodes. The inner cylinder, or shield, of radius 5.5 mm., was drilled with 20 holes 1 mm. in diameter. The next cylinder, or grid, of radius 6.7 mm. had 20 holes, 3 mm. in diameter, accurately centered over the smaller holes in the shield. The plate or outer cylinder was of radius 22.2 mm., and had caps over the ends, the center of the caps being drilled with holes having a radius of 7.8 mm. to give a clearance of about 1 mm. between them and the grid. The apparatus was constructed entirely of nickel, the plate and its end caps being lined with nickel gauze to reduce secondary electron emission. The gas pressure was measured by an ionization manometer mounted in the same chamber with the apparatus just described.

In operation, the shield and plate were connected together through a galvanometer and held at a positive potential of from 100 to 2500 volts with respect to the filament, and the grid was held at a negative potential of from 20 to 75 volts with respect to the shield and plate. This negative potential is the field which drives the positive ions back to the grid to be collected and measured there. Most of the electrons given off by the filament strike the shield, but a few pass through the holes in the shield, on through the larger holes in the grid, and strike the plate. The deflection on the galvanometer connected between the plate and shield is proportional to the number of these electrons. If there is gas present, positive ions are produced which are collected on the grid and their number

is proportional to the deflection of the galvanometer between the grid and shield.

Knowing the current of ionizing electrons, the current of positive ions, the dimensions of the apparatus, and the gas pressure, the effective molecular radius can be calculated, for the number of collisions, n_2, of n_1 electrons going through a gas for a distance $R-R_0$ (the difference between the plate and grid radii) is

$$n_2 = n_1 \quad \pi a^2 \quad \frac{PN}{760} \quad (R-R_0)$$

which follows by integration between cylindrical electrodes of the relation given by Van der Bijl[3] for plane electrodes, in which:

a is the effective molecular radius, defined as the radius of a sphere about the molecular center within which an electron must pass to cause ionization;

P is the gas pressure in mm. of mercury;

N is the number of gas molecules per cc. at 760 mm. pressure $= 2.70 \times 10^{19}$.

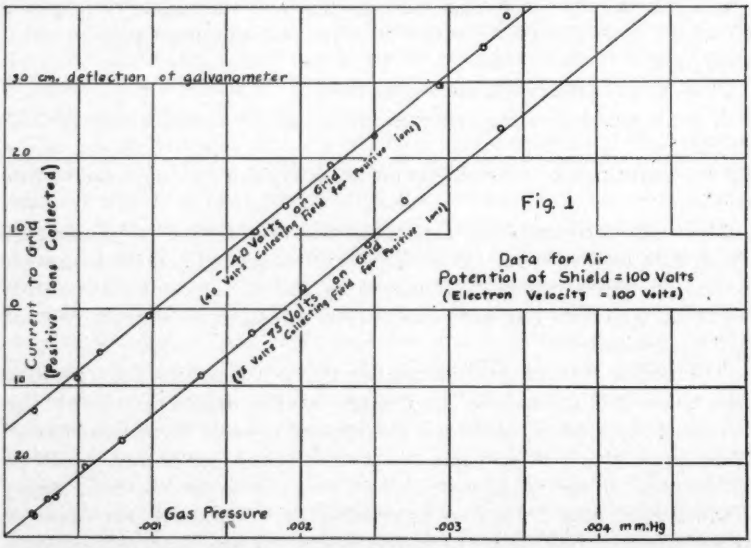
All the quantities in this equation are known except "a," so it can be calculated.

The characteristics of the apparatus were first studied in considerable detail with high vacuum. Then the gas was admitted and the ionization could be determined from the change in the grid current due to the positive ions collected by the grid, and from this change of grid current, the value of "a" was computed.

The method of making this calculation was to plot the current of positive ions to the grid against the gas pressure as read simultaneously by the ionization manometer, as the gas was allowed to enter the system slowly. This should give a straight line, for the number of ions formed should be proportional to the gas pressure. Two such curves are shown in figure 1 which are typical of the data as given by the apparatus. The slopes of these lines give the ratio of the current of positive ions to the gas pressure, or the ratio n_2/P in the relation for calculating "a." The only other variable is n_1, the current of ionizing electrons, which was held constant for each set of data. The constants of the apparatus, (R and R_0), were measured before assembly. Two or three lines, such as shown in figure 1 were plotted for each shield voltage (or electron velocity), the different lines being for different grid voltages, corresponding to different fields drawing the positive ions to the grid, but usually having very nearly equal slopes. From the slopes of these lines, the values of "a" were calculated, giving the effective molecular radius for electrons of the velocity corresponding to the potential of the shield.

It must be recognized that the situation inside the tube is necessarily quite complicated. There are some secondary electrons, and these may

contribute to the ions formed. But, from a careful study of the characteristics of the apparatus in high vacuum, it was shown that most of the secondary electrons were of low velocity, and would be quickly turned back to the plate at the potentials used, and, therefore, their effect was probably small. Also the secondary electron emission of the gauze covered surface was shown to be much smaller than would be expected from a pure nickel surface. It has been suggested that the electron stream may spread out after entering the ionization chamber and not conform to the conical form, determined by the geometry of the holes through which it passes, and which was assumed in integrating to get the equation used in the calculations. Such a spreading would have no effect on the calculations



until the deviation from a radial path was great enough so that the electron path through the gas was appreciably lengthened. At the voltages used, the spread would be very small, and would cause no large error.

From simple considerations, the curves of figure 1 should go through the origin. The displacement is not due to a leak, for there was no grid current until the filament was heated. It was probably due partly to electrons getting to the grid from the comparatively dense electron atmosphere inside the shield and seems to be due also to effects from residual gases occluded in the metal parts, as the position of the lines depended on the degree to which the tube had been baked out. Since the intercept at zero gas pressure occurred at both positive and negative values of grid current, there must be two effects acting, one contributing stray electrons

to the grid current and another giving positive ions. The latter effect is more prominent at higher voltages, for the current value at the intercept increased regularly with increasing voltage. At low voltages, probably more stray electrons from the dense electron atmosphere around the filament reach the grid, giving reversed grid currents at zero gas pressure

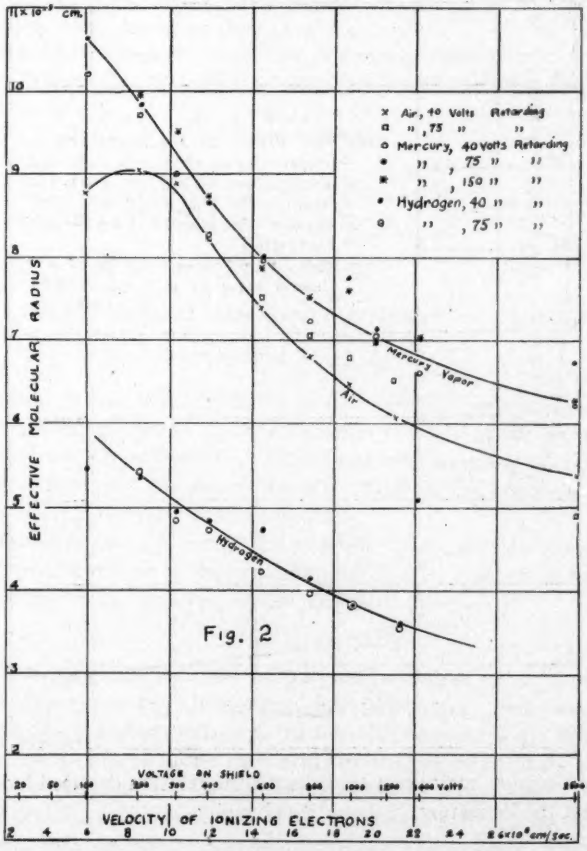
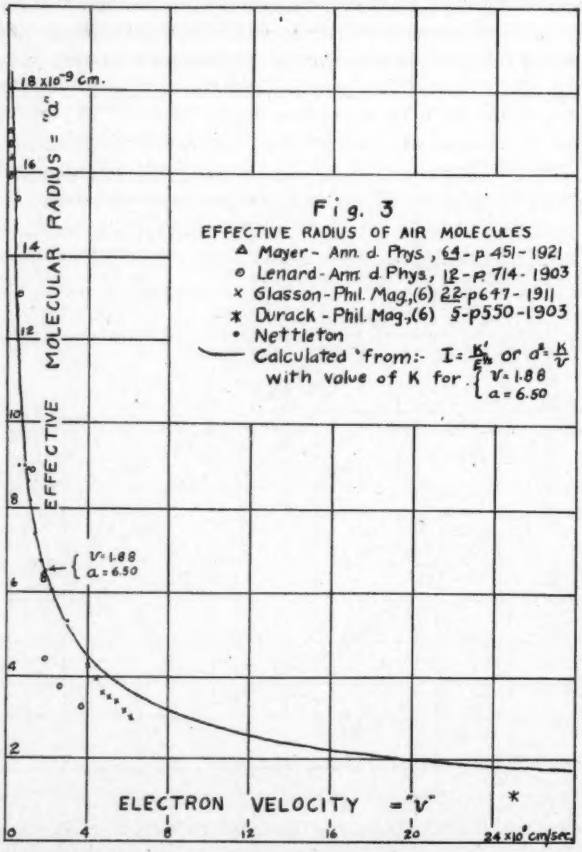


Fig. 2

for the lower voltages used. Since the method is essentially one of taking differences, the results are not effected unless these effects change appreciably when a small amount of gas enters the system.

Measurements were made for air, hydrogen, and mercury vapor over a range of electron velocities from 100 to 2500 volts. The results are shown by the curves of figure 2, where the effective molecular radius is plotted against the electron velocity. It is seen that for the three gases,

studied, there is a regular decrease of the effective molecular radius with increasing electron velocity. This suggests that the change in the ionizing



power of electrons may be in accord with the relation deduced by Rutherford,[4] that the ionization, I, should be given by

$$I = \frac{K'}{E^{1/2}}$$

where E is the energy of the electrons and K' a constant. The corresponding relation in terms of electron velocity, v, and molecular radius, a, is

$$a = \frac{K}{V}$$

In order to test this further, the data of other workers who have measured ionization in air due to electrons, cathode rays and Beta rays were reduced

to the basis of effective molecular radius. The results are shown in figure 3. The full curve is calculated, choosing a value of K to fit the point indicated by the arrow in the author's data. It is seen that over a range of velocities from 10^8 to 2.5×10^{10} cm. per sec., the ionization is in general agreement with Rutherford's formula.

[1] Barber, *Physic. Rev.*, **17**, 1921, p. 326.
[2] McAllister, *Ibid.*, **21**, Sept., 1922, p. 137.
[3] Van der Bijl, *Thermionic Vacuum Tube*, first edition (1920), p. 91.
[4] Rutherford, *Radio-Active Substances and their Radiations* (1913), p. 240.

NOTE ON ABSORPTION MEASUREMENTS OF THE X-RAYS REFLECTED FROM A CALCITE CRYSTAL

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This research was undertaken (1) to determine by an accurate absorption measurement whether there is a change of wave-length on reflection of X-rays from a crystal, and (2) to find out whether there exists any appreciable difference in the absorption of an X-ray of the same wave-length before and after reflexion from a crystal.

Measurement of the Change of Wave-Length.—According to Prof. Compton's[1] quantum theory of the scattering of X-rays, there is a change of wave-length due to scattering whose magnitude is

$$0.0242\,(1 - \cos\theta)A.U. \qquad (1)$$

where θ is the angle between the primary and scattered ray. If reflection of X-rays from crystals is a scattering phenomenon, it would seem that there might be a similar increase in wave-length on reflection. The corresponding change in absorption coefficients for the primary and the reflected ray should be detectable if the scattering angle θ is fairly large.

In the present experiment the writer employed a Bragg spectrometer and a Coolidge tube with a molybdenum target. A current of about one milliampere passed through the tube coming from a generating plant consisting of transformers and a kenotron so arranged as to produce an intermittent direct current of 50,000 peak volts.

The radiation used was that of the $K\alpha_1$ line of molybdenum reflected in the fourth order from calcite, which corresponded to an angle of scattering equal to about 55°. The corresponding change of wave-length calculated according to equation (1) is $\delta\lambda = 0.0105\,A.U.$ A piece of aluminium

sheet of 0.05 cm. in thickness was used as absorber. The absorber was first placed at a point A, midway between the target and the crystal, and then transferred to B, midway between the crystal and the ionization chamber. Since the primary voltage of the transformer was supplied from the mains, small variations of potential across the tube were unavoidable. Also the intensity of the radiation was found to change gradually due to shift of the target spot on heating in the first few minutes after operation. To make these effects as small as possible, the data were taken in a continuous series of observations lasting over a considerable time. In this way the errors tended to eliminate themselves in the mean of several observations.

The results of the measurements are as follows:

μ_A/ρ	μ_B/ρ	EXPTL. DIFFERENCE	DIFFERENCE CALCULATED
5.275	5.278	$0.003 \pm .003$	0.32

where μ_A/ρ and μ_B/ρ represent the mass absorption coefficients at A and B, respectively. From the data it is seen that within the probable experimental error the mass absorption coefficients are the same for the primary and the reflected ray. Thus the wave-length of X-rays undergoes no change on reflection from a crystal, which is comparable with that expressed by equation (1).

The negative result is in agreement with those obtained by Jauncey and Eckart[2] by spectroscopic measurement and by Kulenkampff[3] by photographic method, though these authors scattered a primary beam of greater wave-length and at a smaller angle than that used in the present work.

The deduction of equation (1) is based upon the assumption that each quantum is scattered by a single electron. The theoretical change of wave-length is inversely proportional to m, where m is the mass of the scattering body. If, then, in the case of crystal reflection, interference indicates that the rays are scattered simultaneously by a group of electrons, the change of wave-length should be negligible.[4]

2. *Measurement of the Change of Absorption Coefficient.*—Prof. Compton[5] has shown that if the absorption of X-rays is to be accounted for according to the classical electrodynamics, there is a term which should appear in the expression for the total absorption in addition to the "true absorption" and the scattering. He finds on certain assumptions that the atomic absorption coefficient due to this "transformed" radiation is of the same order of magnitude as the scattering coefficient σ_0/ν calculated from J. J. Thomson's theory, the expression for this atomic "transformation coefficient" being approximately given by

$$\alpha/\delta = R'\tau\sigma_0/\nu, \qquad (2)$$

where R' is a quantity of the order of magnitude comparable with but less than unity and τ is a constant depending upon the amount of resonance.

The argument leading to equation (2) is based upon the assumption that the incident wave begins suddenly at the time $t = 0$. An electron excited by this wave-train will be set into a forced oscillation which will be composed of two vibrations, one having the frequency of the incident wave and the other corresponding to the free oscillation of the electron. The first type of oscillation results in the scattering, while the second type of oscillation produces the "transformed radiation" in question. If, however, instead of starting suddenly the amplitude of the incident wave gradually increases to a maximum and then dies down, as in the case of wave-train reflected from a crystal, the energy thus transformed into the natural vibration of the absorbing electron is very small. It can be shown[6] that the atomic transformation coefficient due to a wave-train reflected from a perfect crystal composed of n layers of atoms equally spaced and of similar composition is a negligible fraction of that for the beam before it was reflected from the crystal.

It, therefore, might be possible to detect this transformation absorption by measuring the total absorption of radiation of the same wave-length before and after reflextion from a crystal.

Previously Moseley and Darwin[7] had found that the intensity of reflection from a crystal was cut down by exactly the same amount by a given piece of aluminium, whether it was placed so as to intercept the whole radiation before reflexion, or the selected radiation after reflexion. But it might happen that the difference in absorption under consideration is so small that it might be beyond the means of detection at these authors' disposal.

In performing this measurement the writer used the same apparatus as described in section (1) of the present note. The radiation used was that of $K\alpha$ of molybdenum in the first order. The X-ray tube was operated at a potential of 50,000 to 70,000 volts. The absorption was measured at both position A and B.

The final results of this experiment are as follows:

ABSORBER	μ_A/ρ	μ_B/ρ	DIFFERENCE
AL	5.210	5.207	0.003 ± 0.005

where μ_A/ρ and μ_B/ρ represent the mass absorption coefficients at A and B, respectively. It seems that the coefficients obtained for both A and B are the same within experimental error. The existence of absorption due to "transformation" is, therefore, not detected by the present experiment.

In conclusion the writer wishes to express his gratitude to Prof. A. H. Compton, under whose direction the work was carried out.

[1] A. H. Compton, *Physic. Rev.*, May, 1923.

[2] G. J. M. Jauncey and C. H. Eckart, *Nature*, **112**, No. 2809, Sept., 1923.

[3] H. Kulenkampff, *Zeit. Physik*, 19 Band, 1 Heft, Oct., 1923.

[4] A. H. Compton, *Phil. Mag., London*, **46**, Nov. 1923, p. 910.
[5] A. H. Compton, *Washington Univ. Studies*, **8**, 1920–1921.
[6] A. H. Compton, *loc. cit.*
[7] H. G. J. Moseley and C. G. Darwin, *Phil. Mag.*, **26**, 1913.

THE SECONDARY AND TERTIARY RAYS FROM CHEMICAL ELEMENTS OF SMALL ATOMIC NUMBER DUE TO PRIMARY X-RAYS FROM A MOLYBDENUM TARGET

BY GEORGE L. CLARK,[1] WM. DUANE, AND W. W. STIFLER

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Spectrometric experiments described by Clark and Duane in four notes[2] prove that the X-rays from a secondary radiator due to primary tungsten rays contain: (1) scattered rays with wave-lengths equal to those in the primary beam (both the K and the L series of tungsten); (2) fluorescent rays with wave-lengths equal to those in the primary spectra of the chemical elements in the radiator; and (3) tertiary rays, produced by the impact of K and L photo-electrons on neighboring atoms. According to these experiments the short wave-length limits of the tertiary rays from various radiators agree closely with the equation

$$\lambda = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}, \qquad (1)$$

derived from the theory which gives the kinetic energy of the photo-electrons. In this equation, λ represents the short wave-length limit of the tertiary rays, λ_1, the primary wave-length and λ_2, a critical absorption wave-length characteristic of a chemical element in the secondary radiator.

Continuing these researches, we have used primary rays from a water-cooled molybdenum target tube and secondary radiators consisting of chemical elements of low atomic number—lithium, graphite, ice, rock-salt aluminium and sulfur. The wave-lengths of the K series lines of molybdenum lie in the neighborhood of .7 Ångström; whereas those in the K series of tungsten lie in the neighborhood of .2 Ångström. The notes referred to describe the experimental arrangement. The storage battery and the generator-transformer-kenotron system in parallel with each other supplied a current through the tube of 16 milliamperes at 38,000 volts. The weakness of the currents in the ionization chamber of the spectrometer necessitated the use of somewhat wider slits than in the

previous experiments with a tungsten-target-tube. The widths of the two slits between the secondary radiator and the calcite analyzing crystal were measured in order to make proper corrections in estimating the dis-

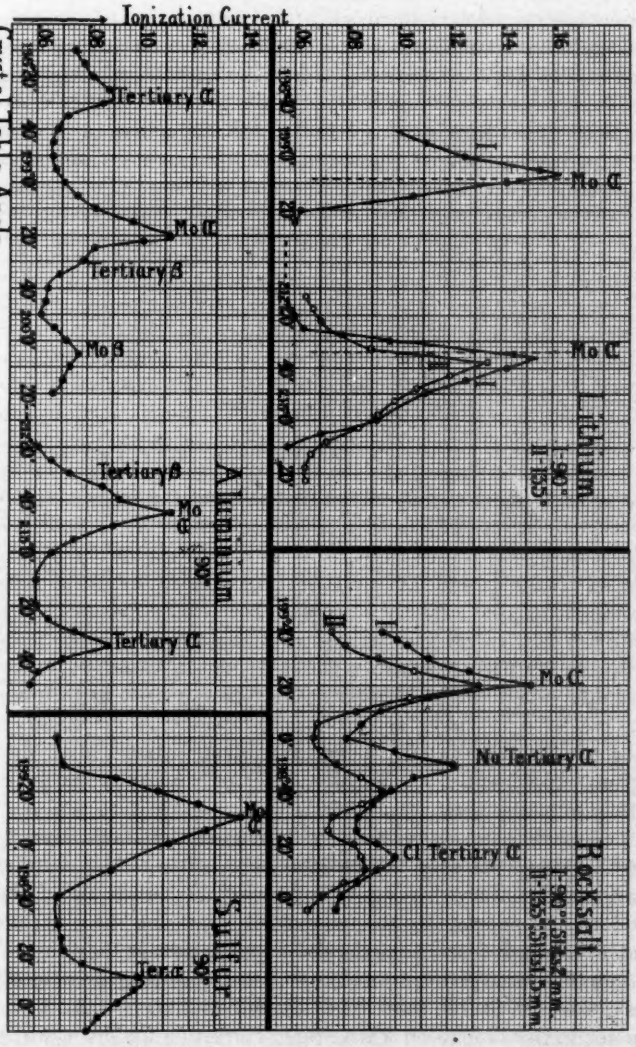

FIGURE 1

placement of the *limits* of the tertiary radiation. In order to estimate the true position of the limit of the tertiary radiation on the crystal table

angle scale one must add a certain slit correction to the angle corresponding to the point where the curve begins to rise. This correction equals half the sum of the widths of the two slits, divided by the distance between them, the ratio being expressed in angular measure. Similarly, in order to calculate where the tertiary hump ought to begin to rise, one should add to the angle corresponding to the top of the peak representing the undisplaced ray the angle $\Delta\theta$ corresponding to the wave-length difference $\Delta\lambda = \lambda - \lambda_1$ (from equation 1) and then subtract from this sum the above slit correction. The distance between the slits amounted to 500 mm. In some experiments (lithium, aluminium) each slit had a width of 1.5 mm., and in others (ice, rock-salt, sulfur), a width of 2 mm. The corresponding slit corrections are, therefore, 10′20″ and 13′45″, respectively. Figure 1 shows some of the results for lithium, rock-salt, aluminium and sulfur as secondary radiators.

We have performed the following experiments with the results mentioned:

1. *Lithium.*—Two complete experiments with angles of 90° and 135° between the primary and the secondary rays, each experiment with readings on both sides of the direct beam of the spectrometer, which establishes an accurate zero. The curves show a somewhat unsymmetrical *single peak* in every case. In the 90° experiments, the top of the peak lies at an angle about 2′ greater than that at which the unshifted line usually appears, and in the 135° experiments about 4′ larger. There is no evidence of peaks which, according to A. H. Compton's theory, should lie at about 14′ (at 90°) and 22′ (at 135°) from the unshifted α peaks. On the other hand, the value of $\lambda - \lambda_1$ for the tertiary radiation is so small for lithium as to merge the tertiary band completely with the unmodified $K\alpha$ peak. Its actual existence, however, is indicated by three facts: (1) comparatively great intensity; (2) apparent shift in the top of the peak, greater with larger angle between primary and secondary beams, analogously to the shift in the tertiary hump *maxima* observed in our tungsten experiments; (3) smaller slope and bulge on the long wave-length side of the peak.

2. *Graphite.*—One experiment with readings on both sides of zero, at 90°. Our results with this radiator agree with A. H. Compton's well known experiments with graphite and paraffin. Two peaks of about the same intensity appear, one the unmodified α line and the other displaced about 16′ towards larger angles. While this is greater than the 14′ found by Compton this experiment may be construed as being favorable to either theory, since $\Delta\lambda$ for the short wave-length limit agrees with its calculated value for tertiary radiation when proper correction for slit width is made, at least as to order of magnitude.

3. *Ice.*—An experiment at 90°. The second peak is displaced 19′ from

the first, 5′ more than expected upon the quantum theory; but here again $\Delta\theta$, for the short wave-length limit, agrees with the value 11′30″ calculated for the tertiary band, as nearly as can be estimated.

4. *Aluminium.*—Two complete experiments at 90° on both sides of zero and including also the β line. The tertiary radiation peak appears with a maximum about 50′ from the $K\alpha$ peak. The displacement of the limit corrected for slit widths agrees almost exactly with that calculated for tertiary radiation, about 40′. In addition, there is evidence of humps on the short wave-length sides of the α peaks produced by tertiary radiation due to the molybdenum $K\beta$ line.

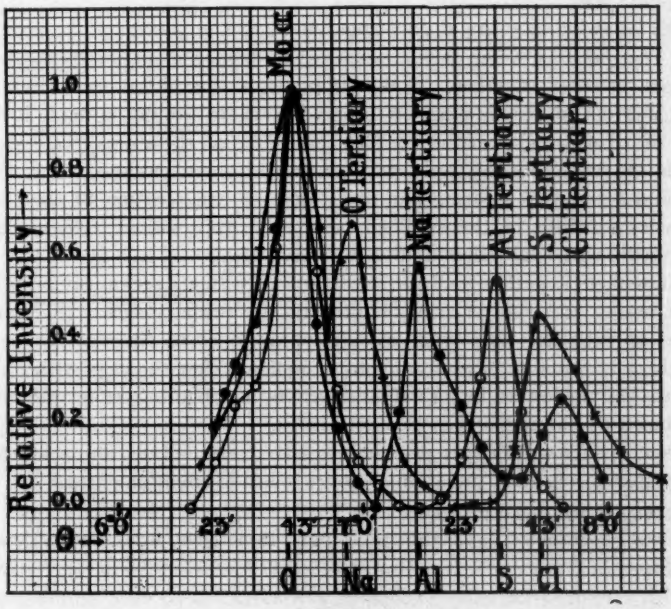

FIGURE 2

5. *Sulfur.*—An experiment at 90° in which the second hump is still further displaced than in the case of aluminium. $\Delta\theta$ is approximately 67′ and agrees with the value calculated from the tertiary radiation equation.

6. *Rock-salt.*—Two complete experiments at 90° and one at 135°. Here three peaks appear, one the $K\alpha$, the next the tertiary radiation from sodium and the third that from chlorine. There is no evidence of a line shifted in accordance with the theory of the transfer of quanta to single electrons. The short wave-length limits agree approximately with those calculated for sodium 26′ 20″, and for chlorine 76′ and these are the same

in the experiments at both 90° and 135°, though the maxima shift over in the latter case.

Figure 2 is a composite diagram showing the results for ice, rock-salt, aluminium and sulfur reduced to the same scale and with the $K\alpha$ line at the same point. The ordinates represent the heights of the peaks only. The theoretical positions for the points where the tertiary humps begin to rise are indicated at the bottom of the diagram. They do not differ from the positions estimated for the curves by more than about two per cent of λ. This is as close as can be reasonably expected in view of the size of the currents measured, the number of points plotted, the intermingling of the peaks, etc.

The following facts are at once apparent: (1) The *position of the modified peak is a function of the atomic number*, the displacement increasing with increase in atomic number; (2) the intensities of the shifted peaks decrease with increase in atomic number (proper allowance should be made in the case of ice and rock-salt for the fact that while all the atoms coöperate in producing the $K\alpha$ peak, only part produce a given modified peak, O in ice, and Na and Cl separately in rock-salt); (3) the position of the short wave-length limit of the modified peak is in approximate agreement in every case with that calculated for tertiary radiation.

These facts, together with the results of experiments previously reported with longer and shorter wave-lengths in the primary beam (K and L series of tungsten) and with secondary radiators of chemical elements ranging in atomic number from 6 to 60 indicate that the rays with wave-lengths shifted toward larger values should be ascribed to tertiary radiation produced by the impacts of photoelectrons.

Experiments with longer primary rays are contemplated in an attempt to measure accurately the atomic energy levels of small magnitude.

[1] NATIONAL RESEARCH FELLOW.

[2] These PROCEEDINGS, **9**, 413, 419 (Dec., 1923); **10**, 41 (Jan., 1924); **10**, 92 (March, 1924).

DENSITY AND DIFFUSION OF GASES MEASURED BY DISPLACEMENT INTERFEROMETRY

BY CARL BARUS

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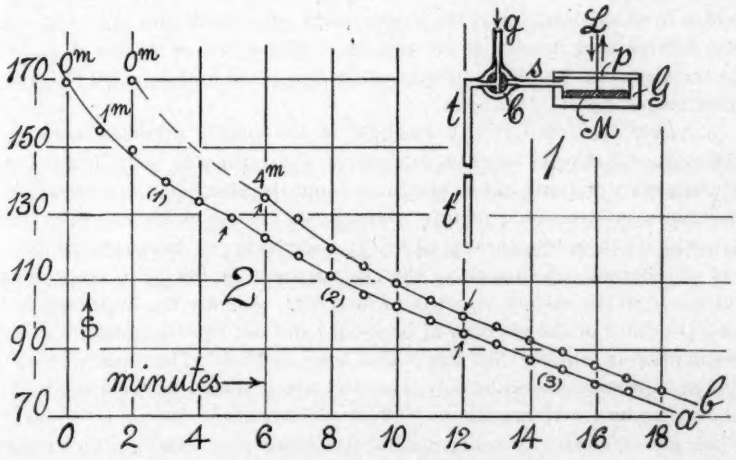
1. *Apparatus.*—The present method is singularly simple and expeditious, and should nevertheless give results well within 1%. I have carried it through preliminarily to exhibit its entire feasibility and to suggest appropriate modifications of apparatus. The plan is virtually a U-tube comparison of the pressure of a rather long column of gas with that of an identical column of air. In figure 1 (diagram) *tt* is the quill tube



(68 cm. long) in which the gas (here coal gas, lighter than air) is to be stored. This communicates by the three-way stopcock *C* with one shank *G* (10 cm. in diam., 1 cm. deep) of the interferometer U-gauge. *M* shows the flat charge of mercury, *p* the sealed cover glass, *L* one component beam of the interferometer. The other shank (not seen) is merely open to the atmosphere.

With *C* set as in figure, the gas arriving at *g* is passed in excess through *tt*. After a few minutes the plug of *C* is suddenly rotated 90° counterclockwise and the pressure produced at *M* read off in fringe displacements in the telescope of the interferometer. The zero of the ocular scale will usually have to be set by the slide micrometer. The operation should be repeated many times to obviate any marked throw of fringes, as the reading of the ocular micrometer must be made (say within 5 sec.), before appreciable diffusion has set in.

If a stopwatch is started when the gas is definitely shut off, the progress of the diffusion phenomena (here occurring at both ends of *tt*) is registered by the displacement of fringes from minute to minute. *C* should be slightly above *M*, the tubulure *s* slanting upward toward the left. The adjustment given is convenient in requiring but little gas and it may also possibly serve for vapor density; but for greater precision it is desirable to so modify the apparatus that the region *GsCt* may all be filled with gas initially and diffusion take place from the bottom of *tt* only.

2. *Equations for Density.*—Let p be the observed pressure, h the mercury head of density ρ_m, x the reading of the slide micrometer normal to mirror, s the reading of the ocular micrometer and θ the angle of reflection of the rays of the quadratic interferometer. Then the equations

$$\Delta h = \Delta x \cos\theta, \qquad \Delta p = \Delta x \cos\theta \rho_m g = H\Delta\rho g$$

follow in succession, if H is the length of the gas tube *tt* and $\Delta\rho = \rho' - \rho$, the difference of density of air and gas. Hence $\Delta\rho = \Delta x \cos\theta\ \rho_m/H$. In the apparatus $\theta = 46.8°$ (provisionally measured) and $\Delta x = 10^{-6}21.5\,\Delta s$. This makes $\Delta\rho = 10^{-6}2.94\,\Delta s$.

3. *Data for Density.*—An example of the results obtained both for diffusion and density is given in figure 2, the ordinates being fringe displacements s proportional to pressures p and the abscissas the corresponding times in steps of 1 minute. One notes the cusps indicated by the arrows with three branches ((1), (2), (3)), which may be due to the diffusion out of different constituents of the gas; or more probably, to the sudden release from the surface viscosity of mercury. As only the beginning and end (15 min.) of the curves will be used, I did not tap the gauge to avoid commotion of fringes; but this is also non-essential. The repeated filling of the tube gave me an initial $\Delta s = 168$ which should be trustworthy to about one unit. Hence $\Delta\rho = 10^{-6} \times 494$ whence, if for air $\rho' = 10^{-6}\cdot 1180$, $\rho = .000686$; or under normal conditions $\rho = .00075$. This result is a pretty close hit in order of value; but for definite results the temperature of the gas in the tube itself should be taken, etc. One may note that if the tube is warmed, the method should include the measurement of vapor densities $\rho/\rho' = 1-(\Delta x\cos\theta/H)(\rho_m/\rho')$, if the last factor is evaluated. Tests made for the coefficient of expansion of the gas in the warmed tube also proved serviceable.

Equations for Diffusion.—If $\partial\rho/\partial t = a^2\partial^2\rho/\partial x^2$ where ρ is the density of the gas at time t and a distance x from the end of the tube, the coefficient sought is a^2 (cm.²/sec.). The problem is virtually that of diffusion in one direction in infinite space, bounded on one side by a plane at which $\rho = 0$ for all time. Initially $\rho = \rho_0$ throughout the whole gas filled space. The general solution was long ago given by Riemann and for the present simple case, if $x/2a\sqrt{t} = y$, leads to

$$\rho = \left(2\rho_0/\sqrt{\pi}\right)\int_0^y e^{-\beta^2}d\beta = \left(2\rho_0/\sqrt{\pi}\right)\left(y - \frac{1}{3}y^3 + \frac{1}{5.2}y^5 - \ldots\right)$$

Since for the vertical tube $dp = \rho g dx$, an integration gives $p = (2x\rho_0 g/\sqrt{\pi})(y/2 - y^3/12 + y^5/60 - \ldots)$. Putting $x\rho_0 g = p_0$ and recalling that $p/p_0 = s/s_0$ and that the tube length of $x = l$, the equation solved for a reads

$$a = \frac{1}{\sqrt{\pi}}\,\frac{s_0}{s}\,\frac{l}{2\sqrt{t}}\left(1 - l/24a^2t + l^4/480a^4t^2 - \ldots\right)$$

seeing that an approximate a sufficies in the corrections. Hence s_0 would be the initial ($t = 0$ sec.) and s the final ($t = 300$ sec.) fringe displacement as given by figure 2; but as in this experiment diffusion takes place at *both* ends of the tube (two tubes each of length $l/2$) the equation becomes finally

$$a = \frac{1}{\sqrt{\pi}}\,\frac{s_0}{s}\,\frac{l}{4\sqrt{t}}\left(1 - l^2/96a^2t + l^4/7680\,a^4t^2 - \ldots\right).$$

Data for Diffusion.—Inserting the data of figure 2

$$t = 0,\ s_0 = 168;\quad t = 300 \text{ sec.},\quad s = 83;\quad l = 68.4 \text{ cm.}$$

the first term is .6508, so that a'^2 would be .4236. The first correction $-l^2/96a^2t = -.1278$; the second correction $l^4/8680a^4t^2 = .0196$ whence $a = .580$ and $a^2 = .337$. This datum is again a hit of the right order of value, but in need of the modifications already instanced. In particular diffusion from both ends not strictly identical is objectionable.

THE BEHAVIOR OF LOW VELOCITY ELECTRONS IN METHANE GAS

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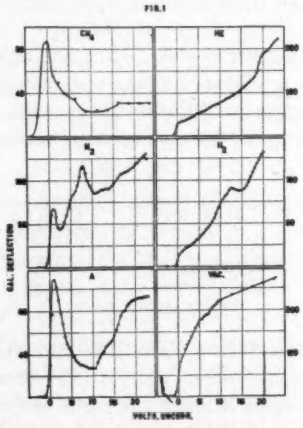
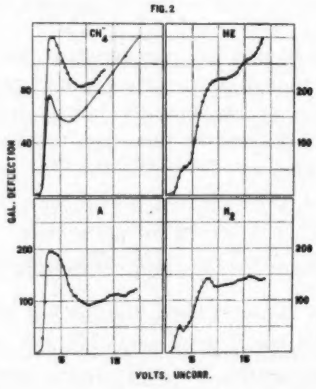
The radiation and ionization potentials of most of the monatomic vapors have been determined[2] experimentally. The natural extension of such research would be the determination of these constants for molecules. The simple molecule hydrogen has been investigated extensively[3] and the results can be reasonably well interpreted in terms of modern atomistics. When atoms and molecules for which no spectroscopic data exists are investigated by the current-potential method, it must be born in mind that not only a radiation-potential but also the phenomenon of trans-

parency as known in argon[4] gives a maximum in the current-potential curves as shown by Minkowski and Sponer in a three-electrode tube. Hence in interpreting results obtained by the current-potential method the occurrence of a maximum cannot be taken as conclusive evidence of a radiation potential.[5]

The specific object of this research is to show that there is a current maximum in methane gas which is not considered to be due to a resonance





potential of the gas, nor its dissociation, nor its thermal decomposition by the filament, but is presumably due to transparency of the methane molecule for electrons of low speed.

Experiments in a Three-Electrode Tube.—Argon, helium, hydrogen, methane and nitrogen were investigated in a three-electrode tube shown in figure 3, and some of the current-potential curves obtained are reproduced in figures 1 and 2. The experiments shown in figure 1 were carried out with a Tungsten filament and those shown in figure 2 were performed with a Platinum-(CaO) filament.

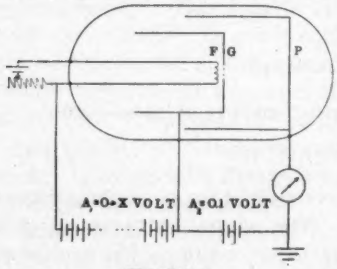


FIGURE 3

Argon[6] prepared by the General Electric Company and helium from the Government were not further purified. Electrolytic hydrogen was freed from oxygen by Pd-asbestos and dried over phosphorus pentoxide. Methane from natural gas was fractionally distilled over liquid air. Nitrogen was made from ammonium nitrite and purified by liquid air and phosphorus pentoxide.

The tube shown in figure 3 has three electrodes: a Tungsten or Platinum-(CaO) filament F; a copper cylinder G, having a 3 mm. hole covered with copper gauze of 31 mesh per inch, at a distance of 1 mm. from the filament and surrounding it; a copper receiving cylinder P with its bottom 2.5 cm. from the gauze and surrounding the other electrodes.

The accelerating field between the gauze G and the cylinder P was kept constant at 0.1 volt and the accelerating field between the filament and gauze G was varied in steps of 0.25 volt.

Discussion of the Experimental Results.—Various pressures and heating currents were employed and a portion only of the data is given in figures 1 and 2; the maxima in the current-potential curves in the case of argon and methane are always very definite. However, no such maxima occurred when hydrogen or helium were in the tube.

By analogy with the interpretation given by Minkowski and Sponer[4] to their results in argon, it is to be inferred that methane also shows the phenomenon of abnormal transparency to low speed electrons.

That the phenomenon in methane is not due to the dissociation of the molecule is probable for the following reason. Dissociation of the methane molecule cannot be caused by electrons, though they have sufficient energy, as is seen from the analogous case of the hydrogen molecule. Three volt electrons do not dissociate the hydrogen molecule though they have sufficient energy. Moreover this is what one would expect from an application of the laws of conservation of energy and momentum to the impact of a three volt electron and hydrogen molecule. By applying the same arguments to the case under consideration it is seen that the maximum observed cannot correspond to the dissociation of the methane molecule.

It was thought possible that the thermal dissociation of the gas by the hot filament might cause the maximum in the current-potential curve. It will be demonstrated below that this was not the case.

The maximum might, however, be due to a radiation-potential. It is seen that it occurs between zero and one volt and would correspond to a band in the red, or to a change in the state of oscillation of the methane molecule. This would mean that an impinging electron can cause this change. It is true that band-spectra of the oscillation-rotation type have thus far been observed in absorption and in emission only by Paschen[7] at high temperatures. However, it is possible that such bands might be excited by electron impact and it cannot be concluded that the phenomenon is not due to a radiation-potential. This must be kept in mind in the present case of methane for which no spectroscopic data exists.

Minkowski and Sponer[4] point out that argon and helium show an increase in current at their radiation potentials of 11.55 and 19.75 volts, respectively. The same effect was also observed in these experiments as is seen from figures 1 and 2 and is presumably due to the abnormal trans-

parency of argon and helium to the low velocity electrons which are produced by collision at the radiation potential. Nitrogen and hydrogen, however, show a decrease at their radiation potentials of 8.2 and 13.5 volts. This would apparently be in conflict with the idea that they show transparency. If these gases have no low radiation potentials, then it would almost seem necessary that the maximum near zero volts in nitrogen and the change in increase of current in hydrogen must be ascribed to a slight transparency of these molecules.

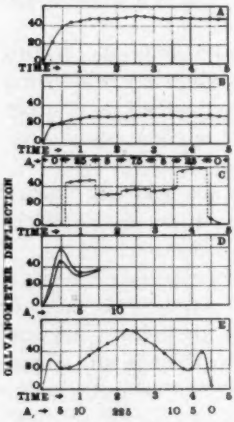


FIG 4

Thermal Decomposition of Methane by the Hot Filament.—It might be supposed that the thermal decomposition of the gas is responsible for the maximum obtained in methane for as hydrogen is created near the filament and carbon is deposited on it in accordance with the reaction $CH_4 \rightarrow C + 2H_2$ the emission of the filament might be affected. Moreover, using the high temperature Tungsten filaments definite evidence of such decomposition was afforded by rise in pressure. With oxide-covered filaments, however, there is no evidence of such decomposition.

This conclusion was further tested with the three-electrode tube in the following manner. The deflections of the galvanometer were noted during five minute intervals with constant accelerating field A_1 and the curves in figure 4 were obtained. It will be noted that with the oxide-covered filament used there is no appreciable change in pressure.

TABLE I

CURVE	A_1 (VOLTS)	HEATING AMPS.	HEATING VOLT	PRESSURE, MM. HG. START	FINISH	REMARKS
A	2.5	1.7	0.87	0.45	0.46	A_1 constant at 2.5 volts
B	5.0	1.7	0.88	0.45	0.46	A_1 constant at 5.0 volts
C	2.5	1.7	0.88	0.45	0.45	A_1 changed to various values as shown
	5.0					
	7.5					
D	2.5	1.7	0.88	0.45	0.45	Data of curve C plotted against A_1
	5.0					
	7.5					
E	variable	1.7	0.87	0.46	0.47	A_1 changed every 15 seconds by 2.5 volts

Curve *A* gives the deflections during five minutes with a constant accelerating field of 2.5 volts from the time the heating current was turned on. Curve *B* similarly gives the deflections with a larger accelerating field of 5.0 volts. It is seen that the current in both runs is quite constant after

the first minute had elapsed, which period was necessary for the filament to get to temperature. Curve *C* gives again the current plotted against time, but with different accelerating voltages as shown in the figure. The applied accelerating voltage is, therefore, shown to have a decided effect on the magnitude of the current. It should be noted that the accelerating voltage was both increased and decreased. Curve *D* shows the galvanometer deflections of Curve *C* plotted against the accelerating voltage. The maximum is clearly brought out. Curve *E* shows a complete run of five minutes duration with the accelerating field increased during the first two and one-half minutes, and decreased the rest of the period. These curves fully demonstrate that the maximum is a function of the accelerating voltage applied to the tube.

Interpretation of Results in a four Electrode Tube.—Special attention should be called to the bearing of the above experiments on the interpretation of the results obtained with four-electrode tubes, when these are used for the determination of radiation-potentials. As a matter of fact, working with a four-electrode tube, an attempt was made to determine the radiation-potentials of methane. It was found, however, that the same maximum obtained with methane in the three-electrode tube was also present in the four-electrode tube, and might easily have been erroneously ascribed to a radiation-potential. It is evident that great circumspection must be used in the interpretation of work with four-electrode tubes.

Summary.—Methane shows a maximum in the current-potential curve obtained in a three-electrode tube. This maximum is shown not to be due to its dissociation, nor its thermal decomposition by the hot filament, but may be due to a low radiation-potential of the gas or its transparency for slow electrons.

By analogy with similar curves in argon, it is thought that the maximum is due to an abnormal transparency of methane for low velocity electrons.

Gases that exhibit the phenomenon of *great* transparency for low velocity electrons show an increase in current at their radiation-potentials, while in others the current decreases at their resonance points.

Conclusions.—A maximum in a current-potential curve as usually obtained when studying resonance potentials may be due to (1) a true resonance point of the gas; (2) abnormal transparency for low velocity electrons. The last point mentioned must be taken into account in interpreting resonance curves.

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[2] Hughes, Research Bulletin, National Research Council, No. 10.

[3] For instance, see Olmstead and Kompton, *Physic. Rev.*, **22**, 559 (1923).

[4] Minkowski and Sponer, *Zt. Physik*, **15**, 403 (1923).

[5] Foote and Mohler, "*The Origin of Spectra*," page 189.

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[7] Paschen, *Ann. Physik*, **53**, 336 (1894).